Basics of Circuit Theory & Network Theorems

1.1: Basics of Network Elements

1.1.1: Electrical Network

A combination of various electric elements (Resistor, Inductor, Capacitor, Voltage source, Current source) connected in any manner whatsoever is called an electrical network. We may classify circuit elements in two categories, passive, and active elements.

1.1.2: Passive Element

The element which receives energy (or absorbs energy) and then either converts it into heat (R) or stored it in an electric (C) or magnetic (L) field is called passive element.

1.1.3: Active Element

The elements that supply energy to the circuit is called active element. Examples of active elements include voltage and current sources, generators, and electronic devices that require power supplies. A transistor is an active circuit element, meaning that it can amplify power of a signal. On the other hand, transformer is not an active element because it does not amplify the power level and power remains same both in primary and secondary sides. Transformer is an example of passive element.

1.1.4: Bilateral Element

Conduction of current in both directions in an element (example: Resistance; Inductance; Capacitance) with same magnitude is termed as bilateral element.

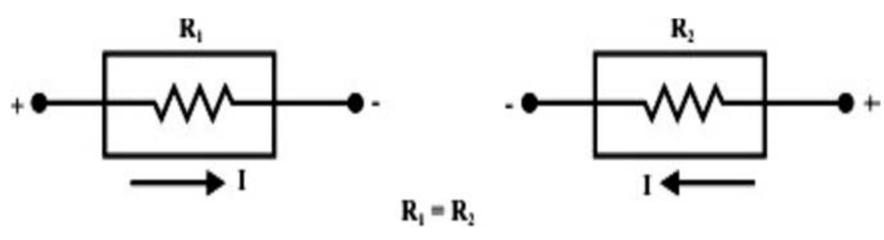


Fig. 1.1 Bilateral element

1.1.5: Unilateral Element

Conduction of current in one direction is termed as unilateral (example: Diode, Transistor) element.

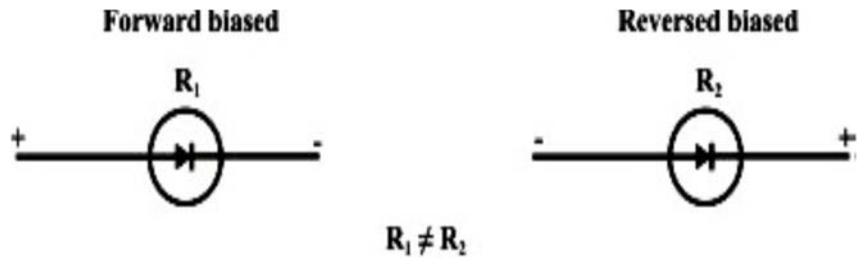


Fig. 1.2 Unilateral element

1.1.6: Meaning of Response

An application of input signal to the system will produce an output signal, the behavior of output signal with time is known as the response of the system.

1.1.7: Linear Circuit

Roughly speaking, a linear circuit is one whose parameters do not change with voltage or current.

1.1.8: Non-Linear Circuit

Roughly speaking, a non-linear system is that whose parameters change with voltage or current. More specifically, non-linear circuit does not obey the homogeneity and additive properties. Volt-ampere characteristics of linear and non-linear elements are shown in Figs. 1.3 – 1.4. In fact, a circuit is linear if and only if its input and output can be related by a straight line passing through the origin as shown in Fig.1.3. Otherwise, it is a nonlinear system.

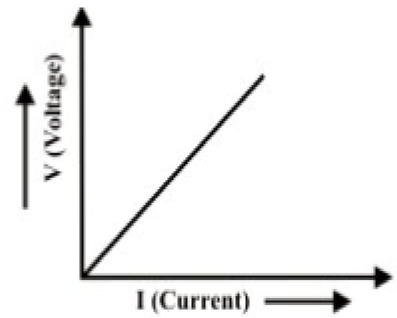


Fig. 1.3 V-I characteristics of linear element

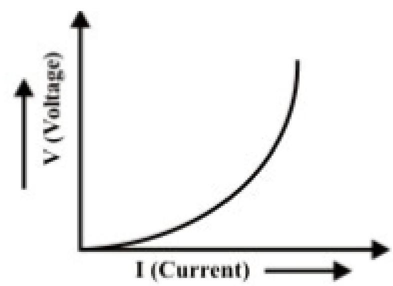


Fig. 1.4 V-I characteristics of non-linear element

1.1.9: Potential Energy Difference

The voltage or potential energy difference between two points in an electric circuit is the amount of energy required to move a unit charge between the two points.

1.1.10: Meaning of Circuit Ground and the Voltages referenced to Ground

In electric or electronic circuits, usually maintain a reference voltage that is named "ground voltage" to which all voltages are referred. This reference voltage is thus at ground potential or zero potential and each other terminal voltage is measured with respect to ground potential, some terminals in the circuit will have voltages above it (positive) and some terminals in the circuit will have voltages below it (negative) or in other words, some potential above or below ground potential or zero potential.

Consider the circuit as shown in Fig. 1.5 and the common point of connection of elements V_1 & V_3 is selected as ground (or reference) node. When the voltages at different nodes are referred to this ground (or reference) point, we denote them with double subscripted voltages V_{AD}, V_{BD}, V_{CD}, V_{ED}. Since the point D is selected as ground potential or zero potential, we can write V_{ED} as V_E , V_{AD} as V_A and so on.

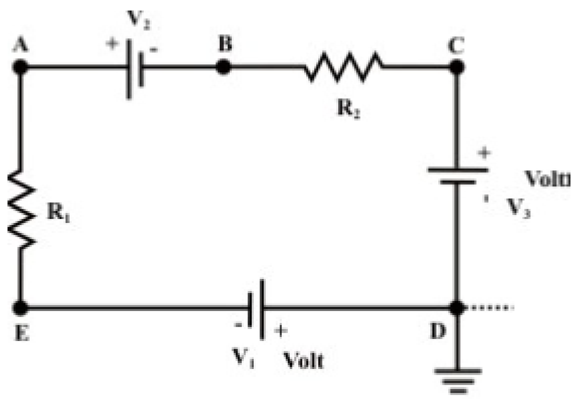


Fig. 1.5 A simple dc resistive circuit

In many cases, such as in electronic circuits, the chassis is shorted to the earth itself for safety reasons

1.1.11: Ideal and Practical Voltage Sources

An ideal voltage source, which is represented by a model in Fig. 1. 6, is a device that produces a constant voltage across its terminals (V= E) no matter what current is drawn from it (terminal voltage is independent of load (resistance) connected across the terminals)

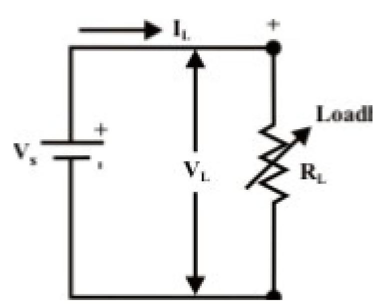


Fig. 1.6 Ideal dc voltage source

For the circuit shown in Fig. 1.6, the upper terminal of load is marked plus (+) and its lower terminal is marked minus (-). This indicates that electrical potential of upper terminal is V_L volts higher than that of lower terminal. The current flowing through the load is given by the expression $V_L=V_S=I_LR_L$ and we can represent the terminal V-I characteristic of an ideal dc voltage as a straight line parallel to the x-axis. This means that the terminal voltage V_L remains constant and equal to the source voltage V_S irrespective of load current is small or large. The V- I characteristic of ideal voltage source is presented in Figure 1.6.

However, real, or practical dc voltage sources do not exhibit such characteristics (see Fig. 1.6) in practice. We observed that as the load resistance R_L connected across the source is decreased, the corresponding load current I_L increases while the terminal voltage across the source decreases (see Eq.1.1). We can realize such voltage drop across the terminals with increase in load current provided a resistance element (Rs) present inside the voltage source. Fig. 1.7 shows the model of practical or real voltage source of value Vs.

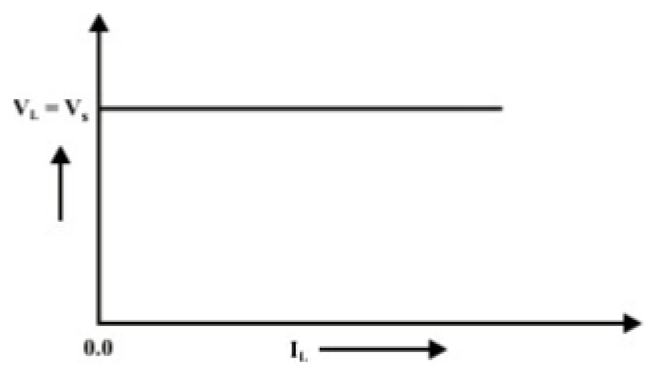


Fig. 1.7 V-I characteristics of ideal voltage source

The terminal V –I characteristics of the practical voltage source can be described by an equation

$$V_L= V_S- I_LR_S \qquad (1.1)$$

and this equation is represented graphically as shown in fig. In practice, when a load resistance more than 100 times larger than the source resistance R_S, the source can be considered approximately ideal voltage source. In other words, the internal resistance of the source can be omitted. This statement can be verified using the relation $R_L=100R_S$ in equation (1.1). The practical voltage source is characterized by two parameters namely known as (i) Open circuit voltage (Vs) (ii) Internal resistance in the source's circuit model. In many practical situations, it is quite important to determine the source parameters experimentally. We shall discuss briefly a method in order to obtain source parameters.

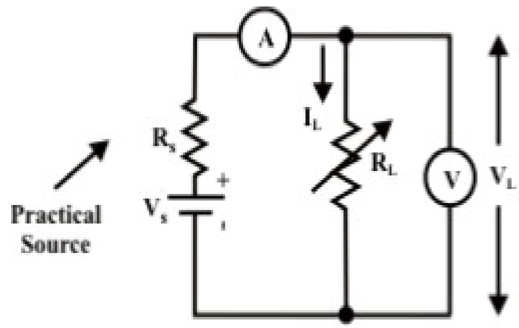


Fig.1.8 Practical DC voltage source model

Method: Connect a variable load resistance across the source terminals (see Fig. 1.8). A voltmeter is connected across the load and an ammeter is connected in series with the load resistance. Voltmeter and Ammeter readings for several choices of load resistances are presented on the graph paper (see Fig. 1.9). The slope of the line is -Rs, while the curve intercepts with voltage axis (at I_L=0) is the value of Vs.

1.1.12: Ideal and Practical Current Sources

Another two-terminal element of common use in circuit modeling is `current source` as depicted in Fig. 1.10(a). An ideal current source, which is represented by a model in Fig. 1.10(a), is a device that delivers a constant current to any load resistance connected across it, no matter what the terminal voltage is developed across the load (i.e., independent of the voltage across its terminals across the terminals).

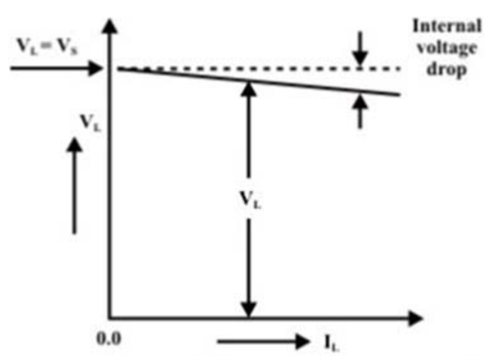


Fig.1.9 V-I characteristics of practical voltage source

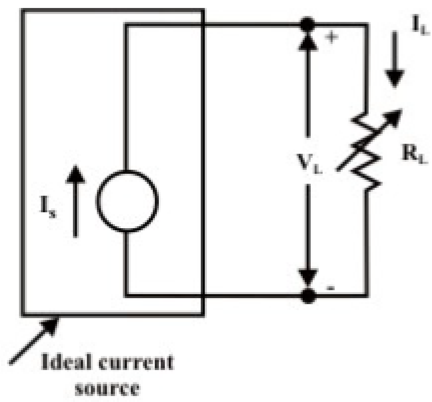


Fig.1.10 (a) Ideal current source with variable load

It can be noted from model of the current source that the current flowing from the source to the load is always constant for any load resistance (see Fig.1.10 (a)) i.e. whether R_L is small (V_L is small) or R_L is large (V_L is large). The vertical dashed line in Fig.1.9 represents the V-I characteristic of ideal current source. In practice, when a load is connected across a practical current source, one can observe that the current flowing in load resistance is reduced as the voltage across the current source's terminal is increased, by increasing the load resistance. Since the distribution of source current in two parallel paths entirely depends on the value of external resistance that connected across the source (current source) terminals. This fact can be realized by introducing a parallel resistance Rs in parallel with the practical current source Is, as shown in Fig.1.10 (b). The dark lines in Fig.1.9 show the V –I characteristic (load-line) of practical current source. The slope of the curve represents the internal resistance of the source. One can apply KCL at the top terminal of the current source in Fig.1.10 (b) to obtain the following expression.

$I_L = I_S - V_L/R_S$ or, $V_L = I_S R_S - R_S I_L = V_{OC} - R_S I_L$ (1.2)

The open circuit voltage and the short-circuit current of the practical current source are given by $V_{OC} = I_S R_S$ and $I_{short} = I_S$ respectively. It can be noted from the Fig.3.18 that source 1 has a larger internal resistance than source 2 and the slope the curve indicates the internal resistance sR of the current source. Thus, source 1 is closer to the ideal source. More specifically, if the source internal resistance $R_S \geq 100 R_L$ then source acts nearly as an ideal current source.

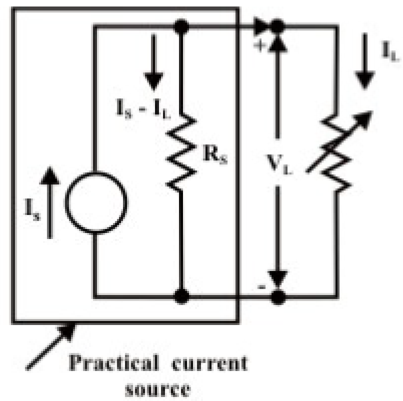


Fig.1.10 (b) Practical current source with variable load

1.1.13: Independent Sources

So far, the voltage and current sources (whether ideal or practical) that have been discussed are known as independent sources and these sources play an important role to drive the circuit in order to perform a specific job. The internal values of these sources (either voltage source or current source) – that is, the generated voltage Vs or the generated current Is (see Figs.1.8 & 1.10 (a & b) are not affected by the load connected across the source terminals or across any other element that exists elsewhere in the circuit or external to the source.

1.1.14: Dependent Sources

Another class of electrical sources is characterized by dependent source or controlled source. In fact, the source voltage or current depends on a voltage across or a current through some other element elsewhere in the circuit. Sources, which exhibit this dependency, are called dependent sources. Both voltage and current types of sources may be dependent, and either may be controlled by a voltage or a current. In general, dependent source is represented by a diamond shaped symbol as not to confuse it with an independent source. One can classify dependent voltage and current sources into four types of sources as shown in Fig.1.11 These are listed below:

(i) Voltage-controlled voltage source (VCVS) (ii) Current-controlled voltage source (ICVS) (iii) Voltage-controlled current source (VCIS) (iv) Current-controlled current source (ICIS)

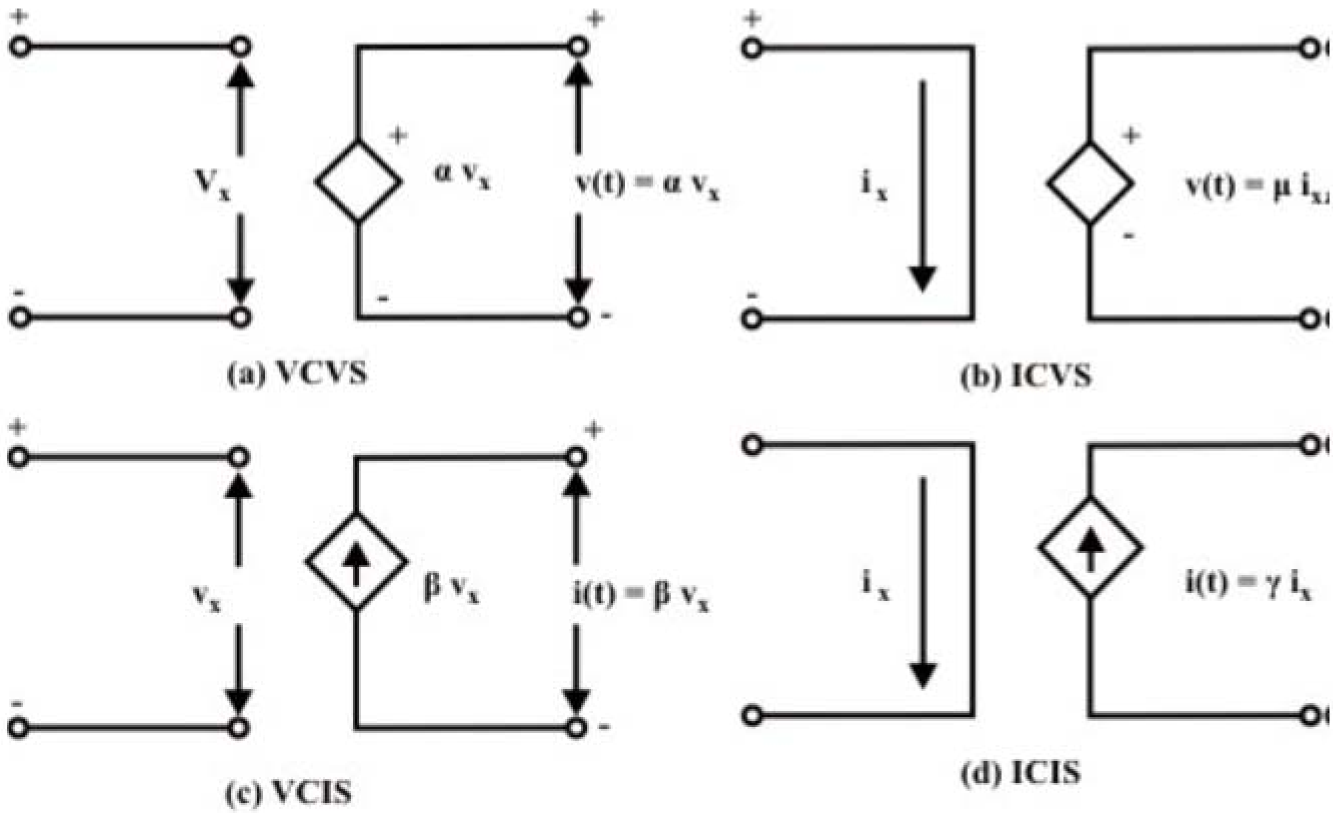


Fig. 1.11 Ideal dependent (controlled) sources

Note: When the value of the source (either voltage or current) is controlled by a voltage (v_x) somewhere else in the circuit, the source is said to be voltage-controlled source. On the other hand, when the value of the source (either voltage or current) is controlled by a current (i_x) somewhere else in the circuit, the source is said to be current-controlled source. KVL and KCL laws can be applied to networks containing such dependent sources. Source conversions, from dependent voltage source models to dependent current source models, or visa-versa, can be employed as needed to simplify the network. One may come across with the dependent sources in many equivalent-circuit models of electronic devices like BJT (bipolar junction transistor), FET (field-effect transistor) etc. and transducers.

1.2 Kirchhoff's Laws

1.2.1: KCL

The algebraic sum of currents at any node of a circuit is zero.

As per KCL,

$i1+i2-i3-i4-i5 = 0$ (1.3)

(the direction of incoming currents to a node being +ve, the outgoing currents should be taken –ve. The reverse sense of directions can also be taken.)

Thus,

$i1+i2 = i3+i4+i5$ (1.4)

i.e the algebraic sum of currents entering a node must be equal to the algebraic sum of the currents leaving a node.

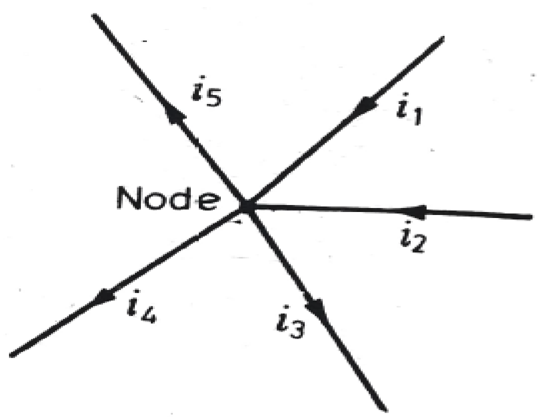


Fig.1.12. Currents meeting at a point in a network.

1.2.2: KVL

The algebraic sum of voltages (or voltage drops) in any closed path of network that is traversed in a single direction is zero.

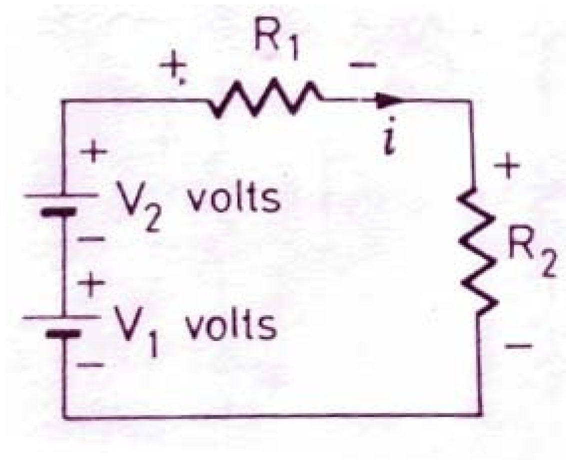


Fig. 1.13

As per KVL,

-V1 +(-V2) + iR1 +iR2 =0 (1.5)

Or,

i (R1 +R2) = V1 +V2 (1.6)

Giving $i=\frac{V1+V2}{(R1+R2)}$

In this context it may be noted that for dependent sources in the circuit, KVL can also be applied. In case of calculation of power of any sources, when the current enters the source, the power is absorbed by the sources while the source delivers the power if the current is coming out of the source.

1.3 Nodal and Mesh analysis of Electric Circuits

There is another method of solution of the network i.e by nodal analysis where it is essential to compute all branch currents. In writing the current expression the assumption is made that the node potential is always higher than the other voltages appearing in the equation. In case it turns out not to be so, a negative value for the current would result. In the nodal method the number of independent node-pair equations needed is one less than the number of junctions in the network i.e , if 'n' denotes the number of independent node equations and 'j' the number of junctions, n= j-1.

There is one more method of analyzing an electrical network – the mesh analysis, the name being derived from the similarities in appearance between the closed loops of a network and a physical 'fence' or mesh. In this method, a distinct current is assumed in the loop and the polarities of drops in each element in the loop is determined by the assumed direction of loop current for that loop. KVL is then applied around each closed loop and by solving these loop equations, the branch currents are determined.

To compare with the mesh method (where KVL equations are frequently used to solve the network), the nodal method is advantageous when the network has many parallel circuits; otherwise, both the nodal and mesh methods offer almost equal advantages [for the mesh method, the number of independent mesh equations needed is m= b – (j -1), where b is the number of branches. If, m< n, the mesh method offers advantages while for m>n i.e., when the number of parallel paths in the network is more, nodal method is preferred.

1.3.1: Matrix Approach of Network Containing Voltage and Current Sources and Reactances, Source Transformation, and Duality

1.3.2: Cramer's Rule for systems of Linear Equation with three variables

Given a Linear System

x-column z-column

↓ ↓

$$
\begin{cases}
a_1x + b_1y + c_1z = d_1 \\
a_2x + b_2y + c_2z = d_2 \\
a_3x + b_3y + c_3z = d_3
\end{cases}
$$

↑ ↑

y-column constant column

Coefficient matrix: D =

$$D = \begin{matrix} a_1 + b1 + c1 \\ a_2 + b2 + c2 \\ a_3 + b3 + c3 \end{matrix}$$

X- matrix: D_x =

$$D_x = \begin{pmatrix} d_1 + b1 + c1 \\ d_2 + b2 + c2 \\ d_3 + b3 + c3 \end{pmatrix}$$

Y- matrix: D_y =

$$D_y = \begin{pmatrix} a_1 + d1 + c1 \\ a_2 + d2 + c2 \\ a_3 + d3 + c3 \end{pmatrix}$$

Z matrix : D_z =

$$D_z = \begin{pmatrix} a_1 + b1 + d1 \\ a_2 + b2 + d2 \\ a_3 + b3 + d3 \end{pmatrix}$$

Solve for X

$$X = \frac{[Dx]}{[D]} = \frac{\begin{vmatrix} d_1 + b_1 + c_1 \\ d_2 + b_2 + c_2 \\ d_3 + b_3 + c_3 \end{vmatrix}}{\begin{vmatrix} d_1 + b_1 + c_1 \\ d_2 + b_2 + c_2 \end{vmatrix}}$$

Example 1.1

Use mesh-current analysis to determine the current lowing in (a)The 5Ω resistance and (b). The 1Ω resistance of the d.c. circuit shown in Figure 1.14

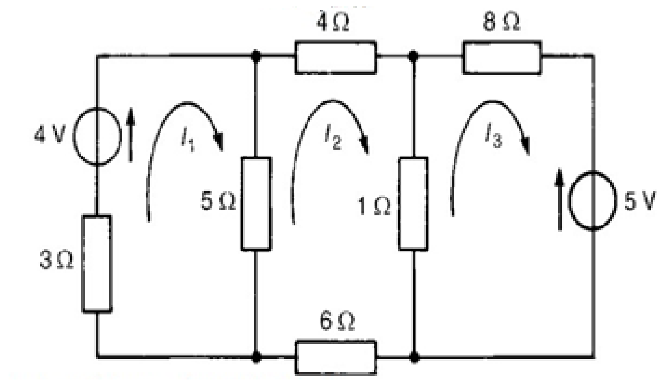


Figure 1.14

The mesh current I_1, I_2 and I_3 are shown in Figure 1.14

Using kirchhoff's voltage law:

$8I_1 - 5I_2 - 4 = 0$

$-5I_1 + 16I_2 - I_3 = 0$

$-I_2 + 9I_3 + 5 = 0$

$$\frac{I1}{\begin{vmatrix} 50-4 \\ 16-10 \\ -195 \end{vmatrix}} = \frac{-I2}{\begin{vmatrix} 80-1 \\ -5-10 \\ 095 \end{vmatrix}} = \frac{I3}{\begin{vmatrix} 8-5-4 \\ -5160 \\ 0-15 \end{vmatrix}} = \frac{I}{\begin{vmatrix} 8-50 \\ -516-1 \\ 0-19 \end{vmatrix}}$$

Using Determinants:

$$\frac{I1}{-5\begin{vmatrix} -10 \\ 95 \end{vmatrix} - 4\begin{vmatrix} 16-1 \\ -19 \end{vmatrix}} = \frac{I2}{8\begin{vmatrix} -10 \\ 95 \end{vmatrix} - 4\begin{vmatrix} 5-1 \\ 09 \end{vmatrix}} = \frac{I3}{-4\begin{vmatrix} -516 \\ 0-1 \end{vmatrix} + 5\begin{vmatrix} 8-5 \\ -516 \end{vmatrix}} = \frac{-I}{8\begin{vmatrix} 16-1 \\ -19 \end{vmatrix} + 5\begin{vmatrix} -5-1 \\ 09 \end{vmatrix}}$$

$$\frac{I1}{-5(-5) - 4(143)} = \frac{-I2}{8(-5) - 4(-45)} = \frac{I3}{-4(5) + 5(103)}$$

$$\frac{I1}{-547} = \frac{-I2}{140} = \frac{I3}{495} = \frac{-I}{919}$$

Hence

$$I_1 = \frac{547}{919} = 0.595\,A.$$

$$I_2 = \frac{140}{919} = 0.152A.$$

$$I_3 = \frac{-495}{919} = -0.539A.$$

(a) Current in the 5Ω resistance = $I_1 - I_2 = 0.595 - 0.152 = 0.44A$

(b) Current in the 1Ω resistance = $I_2 - I_3 = 0.152 - (-0.539) = 0.69A$

1.3.3: Generalized Matrix Representation

The generalized matrix representation of the circuit containing [Z_1,……,Zn] impedances with voltage sources [V_1,…,Vn] to evaluate currents [I_1…In] by mesh equations using KVL is presented in Eq. (1.7)

$$\begin{bmatrix} Z11 Z12 Z13 \cdots Z1n \\ Z21 Z22 Z23 \cdots Z2n \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ Zn1 Zn2 Zn3 \cdots Znn \\ \end{bmatrix} \begin{bmatrix} I1 \\ I2 \\ \vdots \\ \vdots \\ In \end{bmatrix} = \begin{bmatrix} V1 \\ V2 \\ \vdots \\ \vdots \\ Vn \end{bmatrix} \quad (1.7)$$

or, in the compact form

[Z][I] = [V]

Where [Z] is the impedance matrix, and [V] and[I] are the voltage and the current matrices. [Z] is a square matrix whereas [V) and [I] are column matrices as shown in Eq. (1.8)

$$Z = \begin{bmatrix} Z11 Z12 Z13 \cdots Z1n \\ Z21 Z22 Z23 \cdots Z2n \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ Zn1 Zn2 Zn3 \cdots Znn \\ \end{bmatrix} \quad V = \begin{bmatrix} V1 \\ V2 \\ \vdots \\ \vdots \\ Vn \end{bmatrix} \quad \text{and} \quad I = \begin{bmatrix} I1 \\ I2 \\ \vdots \\ \vdots \\ In \end{bmatrix} \quad (1.8)$$

Similarly, the node equations may be expressed as shown in Eq. (1.9)

[Y][V]=[I] (1.9)

where [Y] is the admittance matrix and [V] and [I] are the voltage and the current matrices. The [Y] is a square matrix whereas [V] and [I] are column matrices

$$Y = \begin{bmatrix} Y11 & Y12 & \ldots\ldots\ldots & -Y1n \\ -Y21 & Y22 & \ldots\ldots\ldots & -Y2n \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & & & \\ -Yn1 - Yn2\ldots\ldots\ldots Ynn & & & \end{bmatrix}$$

$$V = \begin{bmatrix} V1 \\ V2 \\ \vdots \\ \vdots \\ Vn \end{bmatrix} \text{ and } I = \begin{bmatrix} I1 \\ I2 \\ \vdots \\ \vdots \\ In \end{bmatrix}$$

1.4 Network Theorems

1.4.1: Superposition Theorem

Objectives

Statement of superposition theorem and its application to a resistive d.c. network containing more than one source in order to find a current through a branch or to find a voltage across the branch.

Superposition Theorem Statement

Any linear, bilateral two terminal network consisting of more than one sources. The total current or voltage in any part of a network is equal to the algebraic sum of the currents or voltages in the required branch with each source acting individually while other sources are replaced by their ideal internal resistances. (i.e., Voltage sources by a short circuit and current sources by open circuit)

Superposition theorem states that

In a linear circuit with several sources the voltage and current responses in any branch is the algebraic sum of the voltage and current responses due to each source acting independently with all other sources replaced by their internal impedance.

OR

In any linear circuit containing multiple independent sources, the current or voltage at any point in the network may be calculated as algebraic sum of the individual contributions of each source acting alone.

The process of using Superposition Theorem on a circuit

To solve a circuit with the help of Superposition theorem follow the following steps:

- First of all, make sure the circuit is a linear circuit; or a circuit where Ohm's law implies, because Superposition theorem is applicable only to linear circuits and responses.
- Replace all the voltage and current sources on the circuit except for one of them. While replacing a Voltage source or Current Source replace it with their internal resistance or impedance. If the Source is an Ideal source or internal impedance is not given, then replace a Voltage source with a short; so as to maintain a 0 V potential difference between two terminals of the voltage source. And replace a Current source with an Open; so as to maintain a 0 Amps Current between two terminals of the current source.
- Determine the branch responses or voltage drop and current on every branch simply by using KCL, KVL or Ohm's Law.
- Repeat step 2 and 3 for every source the circuit has.
- Now algebraically add the responses due to each source on a branch to find the response on the branch due to the combined effect of all the sources.

The superposition theorem is not applicable for the power, as power is directly proportional to the square of the current which is not a linear function.

Steps

1) Select any one source and short all other voltage sources and open all current sources if internal impedance is not known. If known replace them by their impedance.

2) Find out the current or voltage across the required element, due to the source under consideration.

3) Repeat the above steps for all other sources.

4) Add all the individual effects produced by individual sources to obtain the total current in or across the voltage element.

Example 1.2

Find I_a in circuit shown below, where only the current source is kept in the circuit. The 5V is zeroed out yielding a 0V source, or a short. The 9V is zeroed out, making it a short also.

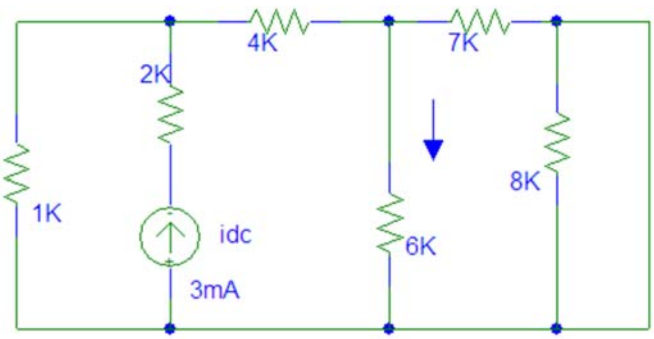


Fig. 1.15

Solution

Note that the 8K resistor is shorted out that is, 8K in parallel with 0 yields 0.

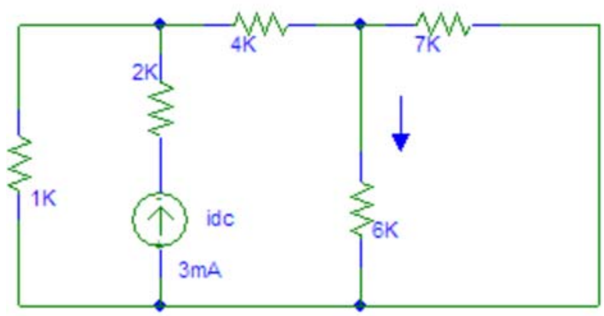


Fig. 1.16

Note that the 3mA flowing up through the 2K will split left and right at the top. Part of it will flow through the 1K and part of it will flow through the 4K. Let's use the label "I4" for the current flowing right through the 4K resistor. If we combine the parallel 6K and 7K (6K||7K = 3.2K) and then add the series 4K, the total resistance on the right is 7.2K. Now we can use a current divider to find that I4 = [1K / (1K + 7.2K)] * 3mA = 0.37mA. Note that the 2K does not enter into this computation because the entire 3mA flows through it. The 3mA does not split until it gets to the junction at the top of that branch. Now that we know I4, we can then split it again through the 6K and the 7K.

Ix = [7K/(6K+7K)]*I4 = 0.20 mA.

Example 1.3

Using the superposition theorem, determine the voltage drop and current across the resistor 3.3K as shown in Fig.1.17 below.

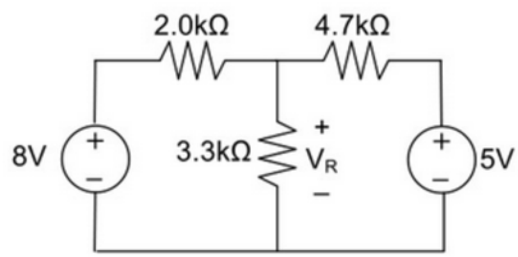


Fig. 1.17

Solution

Step 1: Remove the 8V power supply from the original circuit, such that the new circuit becomes as the following and then measure voltage across resistor.

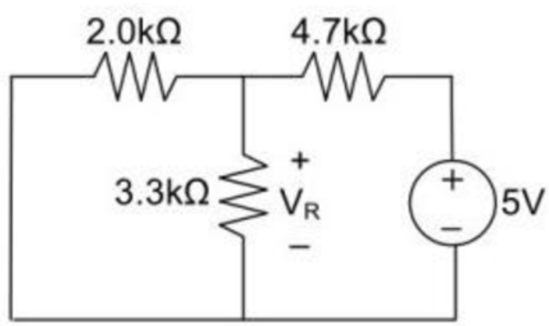


Fig. 1.18

Here 3.3K and 2K are in parallel, therefore resultant resistance will be 1.245K.

Using voltage divider rule voltage across 1.245K will be

V1= [1.245/(1.245+4.7)]*5 = 1.047V

Step 2: Remove the 5V power supply from the original circuit such that the new circuit becomes as the following and then measure voltage across resistor.

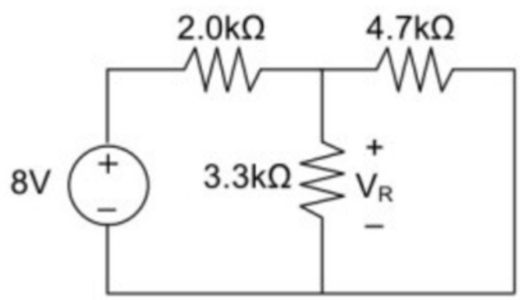


Fig. 1.19

Here 3.3K and 4.7K are in parallel, therefore resultant resistance will be 1.938K.

Using voltage divider rule voltage across 1.938K will be

V2= [1.938/(1.938+2)]*8 = 3.9377V

Therefore, voltage drop across 3.3K resistor is V1+V2 = 1.047+3.9377=4.9847

Example 1.4

By Using the superposition theorem find I in the circuit shown in Fig.1.20.

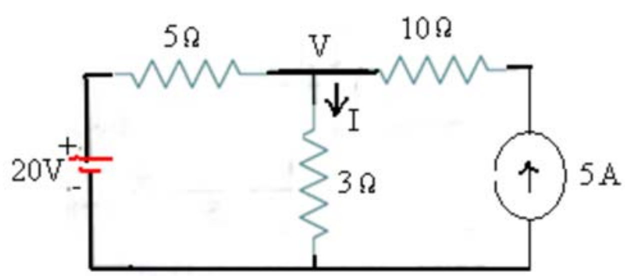
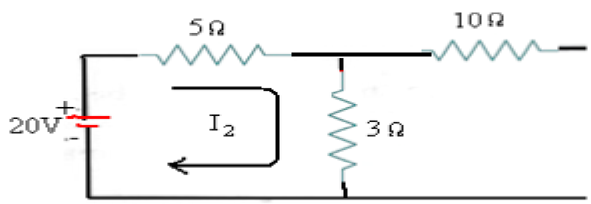


Fig. 1.20

Solution: Applying the superposition theorem, the current **I2** in the resistance of 3 Ω due to the voltage source of

20V alone, with current source of 5A open circuited [as shown in the figure.1.21 below] is given by:



.**Fig. 1.21**

I2 = 20/(5+3) = 2.5A

Similarly, the current I5 in the resistance of 3 Ω due to the current source of 5A alone with voltage source of 20V short circuited [as shown in the below] is given by:

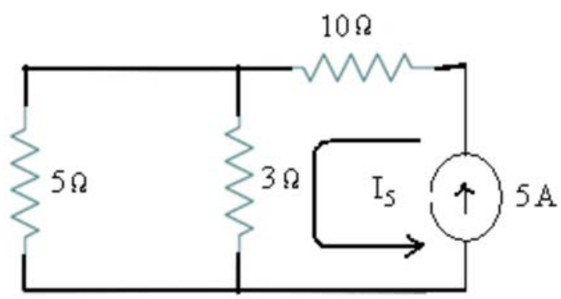


Fig. 1.22

I_5= 5 x 5/(3+5) = 3.125 A

The total current passing through the resistance of 3Ω is then = I2 + I5= 2.5 + 3.125 = **5.625 A**

Let us verify the solution using the basic nodal analysis referring to the node marked with V in fig.1.20. Then, we get: $V-205+V3=5$

3V-60+5V=15×5

8V-60=75

8V=135

V=16.875

The current I passing through the resistance of 3Ω =V/3 = 16.875/3 = **5.625 A.**

Limitations of superposition Theorem

Superposition theorem doesn't work for power calculation. Because **power calculations** involve either the product of voltage and current, the square of current or the square of the voltage, they are not linear operations. This statement can be explained with a simple example as given below.

Example: Consider the circuit diagram as shown in below fig.1.23

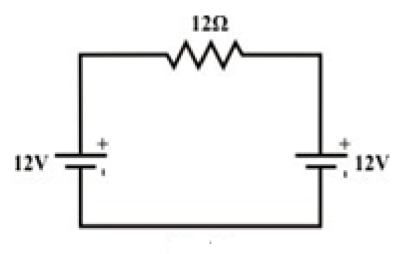


Fig. 1.23

Using superposition theorem one can find the resultant current flowing through 12Ω resistor is zero and consequently power consumed by the resistor is also zero. For power consumed in an any resistive element of a network cannot be computed using superposition theorem. Note that the power consumed by each individual source is given by

$P_{W\ 1(\text{ due to 12V source (left)})}$=12 watts ; $P_{W\ 2(\text{ due to 12V source (right)})}$=12 watts

The total power consumed by 12 Ω = 24 *watts* (applying superposition theorem). This result is wrong conceptually. In fact, we may use the superposition theorem to find a current in any branch or a voltage across any branch, from which power then can be calculated.

- Superposition theorem cannot be applied for nonlinear circuit (Diodes or Transistors).
- This method has weaknesses: -In order to calculate load current I_L or the voltage

V_L for the several choices of load resistance R_L of the resistive network, one needs to solve for every source voltage and current, perhaps several times. With the simple circuit,

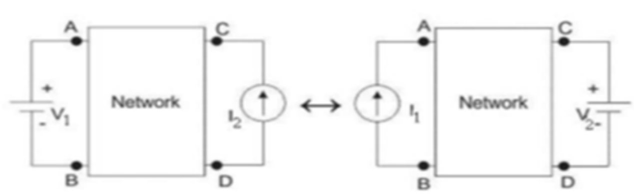


Fig. 1.24.

$V_1 / I_2 = V_2 / I_1$

This is fairly easy but in a large circuit this method becomes an painful experience.

1.4.2: Reciprocity Theorem

Objectives: Statement of Reciprocity theorem and its application to a resistive d.c network by changing the voltage source from branch to branch in order to find a current through a branch.

Reciprocity Theorem Statement

In many electrical networks it is found that if the positions of voltage source and ammeter are interchanged, the reading of ammeter remains the same. Suppose a voltage source is connected to a passive network and an ammeter is connected to other part of the network to indicate the response. Now anyone interchanges the positions of ammeter and voltage source that means he or she connects the voltage source at the part of the network where the ammeter was connected and connects ammeter to that part of the network where the voltage source was connected. The response of the ammeter means current through the ammeter would be the same in both the cases. This is where the property of reciprocity comes in the circuit. The particular circuit that has this reciprocal property, is called reciprocal circuit. Any linear, bilateral two terminal network the ratio of excitation to response is constant even though the source is interchanged from input terminals to the output terminals.

Steps For Solution of a Network Utilizing Reciprocity Theorem

1. The branches between which reciprocity is to be established to be selected first.
2. The current in the branch is obtained using conventional network analysis.
3. The voltage source is interchanged between the branches concerned.
4. The current in the branch where the voltage source was existing earlier is calculated.

It may observe that the currents obtained in 2 & 4 are identical to each other.

Example 1.5

Verify the reciprocity theorem for the given network. Find the current in branch AB

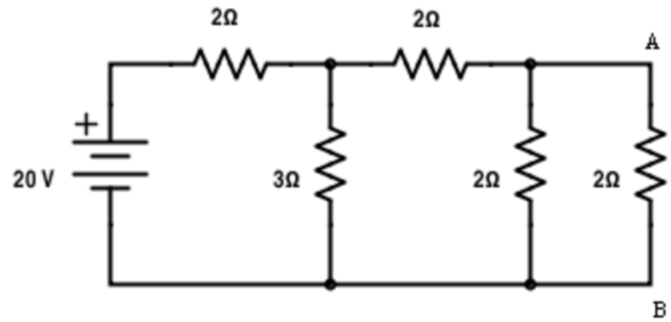


Fig. 1.25

Solution

$R_T = 2+[3 \parallel (2+2 \parallel 2)] = 3.5\Omega$

$I_T = V/RT = 20/3.5 = 5.71A$

Apply current division technique for the circuit to find current through branch AB

= 5.71 x 3/6

= 2.855A

Current in branch AB = 2.855 x 2/4 = 1.43A

Interchanging the source to branch AB

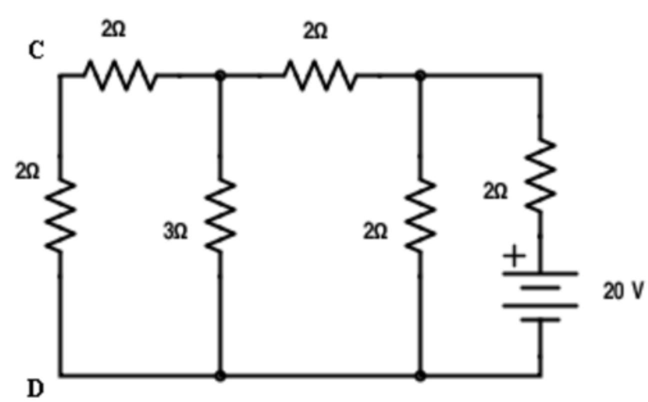


Fig. 1.26

Total resistance $R_T = 3.23\Omega$

I_T = 20/3.23 = 6.19A

Apply current division technique for the circuit to find current through branch CD

= 6.19 x 2/5.2 = 2.38A

Current in branch CD = 2.38 x 3/5 = 1.43A

Hence reciprocity theorem verified.

Example 1.6

Verify the reciprocity theorem for the given circuit diagram.

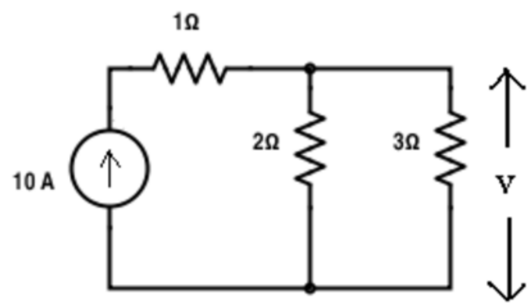


Fig. 1.27

Solution

Apply current division technique for the circuit to find current through branch 3Ω.

= 10 x 2/5 = 4A

Voltage across 3Ω = 4 x 3 = 12V

Replace the current source and find the open circuit voltage

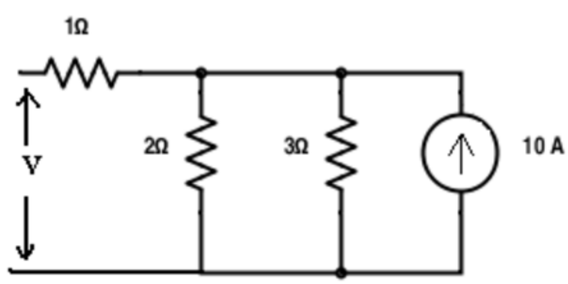


Fig.1.28

Apply current division technique and find the current through 2Ω resistor.

=10 x 3/5 = 6

Voltage across 2Ω resistor = Voltage across AB = 6 x 2 = 12V

Example 1.7

Verify the reciprocity theorem for the network shown in the figure

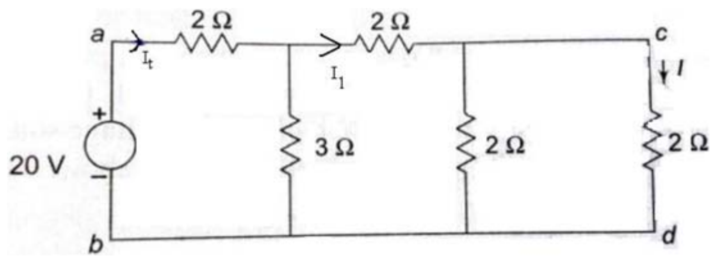


Fig.1.29

Solution

Total resistance in the circuit across the applied voltage of 20 V is

R_{TH}=2 + [3||(2 +(2||2))]

=2 +[3||3]

=3.5 Ω

The total current drawn by the circuit $\mathbf{I_T} = \boldsymbol{V/R_{TH}}$= 20/3.5 = 5.71 A

The current **I** in the branch **'cd'** with 2 Ω resistance is find by using current division rule. For that first find I_1 current.

I_1=5.71×3/(3+3)=2.855A

The current I in the 'cd' branch is

I=2.855×2/(2+2)=1.427A

Now the source voltage and the response are interchanged between branches **‘ab’** and **‘cd’** as shown in the figure below

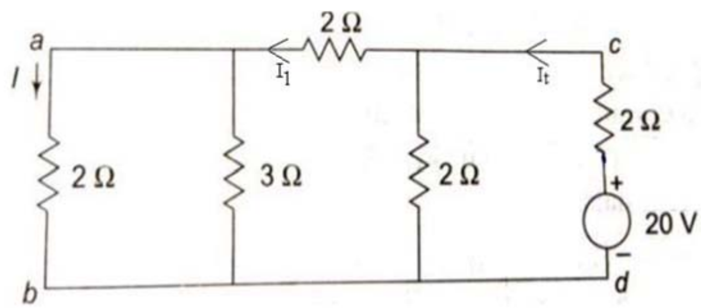


Fig.1.30

Total resistance in the circuit across the applied voltage of 20 V is

R_{TH}=2 + [2||(2 +(2||3))]

=2 +[2||3.2]

=3.23Ω

The total current drawn by the circuit $\mathbf{I_T} = \boldsymbol{V}/\boldsymbol{R_{TH}}$=20/3.23=6.19A

The current **I** in the branch ‘**ab’** with 2 Ω resistance is find by using current division rule. For that first find I_1 current.

I1=6.19×23.2+2=2.38A

The current I in the ‘ab’ branch is

I=2.38×33+2=1.427A

The current in the branch **‘ab’ = 1.427 A** which is same as the current we got in branch **‘cd’** when the voltage was given from branch **‘ab’. Thus** the **reciprocity theorem** is verified.

1.4.3: Thevenin’s Theorem

Objectives:

• To understand the basic philosophy behind the Thevenin’s theorem and its application to solve dc circuits.

• Explain the advantage of Thevenin's theorem over conventional circuit reduction techniques in situations where load changes.

Thevenin's Theorem Statement

Any linear, bilateral two terminal network consisting of sources and resistors(Impedance),can be replaced by an equivalent circuit consisting of a voltage source in series with a resistance (Impedance).The equivalent voltage source $\mathbf{V_{Th}}$ is the open circuit voltage looking into the terminals(with concerned branch element removed) and the equivalent resistance $\mathbf{R_{Th}}$ while all sources are replaced by their internal resistors at ideal condition i.e. voltage source is short circuit and current source is open circuit.

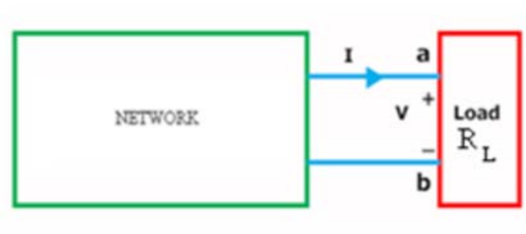


Fig.1.31

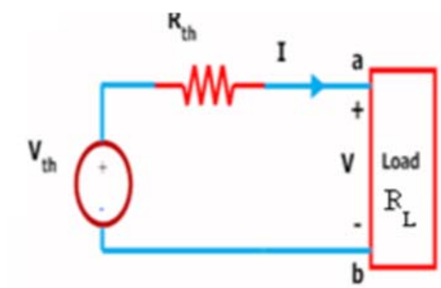


Fig.1.32

Fig.1.31 shows a simple block representation of a network with several active / passive elements with the load resistance $\mathbf{R_L}$ connected across the terminals **'a & b'** and fig.1.32 shows the **Thevenin equivalent circuit** with $\mathbf{V_{Th}}$ connected across $\mathbf{R_{Th}}$& $\mathbf{R_L}$.

Main steps to find out $\mathbf{V_{Th}}$ and $\mathbf{R_{Th}}$

1. The terminals of the branch/element through which the current is to be found out are marked as say **a & b** after removing the concerned branch/element.

2. Open circuit voltage $\mathbf{V_{OC}}$ across these two terminals is found out using the conventional network mesh/node analysis methods and this would be $\mathbf{V_{Th}}$ **.**

3. **Thevenin resistance $\mathbf{R_{Th}}$** is found out by the method depending upon whether the network contains dependent sources or not.

   a. With dependent sources: $\mathbf{R_{Th} = Voc / Isc}$

   b. Without dependent sources: $\mathbf{R_{Th}}$ **= Equivalent resistance looking into the concerned**

terminals with all voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited)

4. Replace the network with $\mathbf{V_{Th}}$ in series with $\mathbf{R_{Th}}$ and the concerned branch resistance **(or)** load resistance across the load terminals(A&B) as shown in below fig.

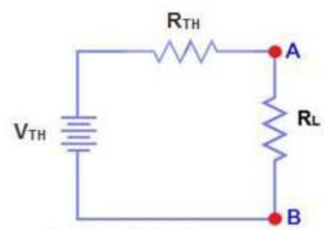


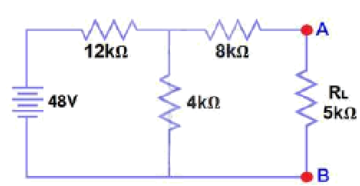
Fig.1.33

Thevenin's equivalent circuit

Example 1.8

Find V_{TH}, R_{TH} and the load current and load voltage flowing through R_L resistor as shown in fig.1.34. by using Thevenin's Theorem.

Fig.1.34



Solution:

The resistance $\mathbf{R_L}$ is removed and the terminals of the resistance $\mathbf{R_L}$ are marked as **A & B** as shown in the fig.1.35

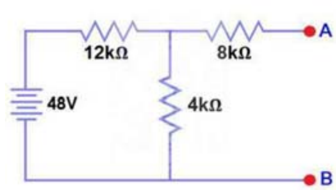


Fig.1.35

Calculate / measure the Open Circuit Voltage. This is the Thevenin Voltage (V_{TH}). We have already removed the load resistor from fig1.34, so the circuit became an open circuit as shown in fig.1.34. Now we have to calculate the Thevenin's Voltage. Since 3mA Current flows in both 12kΩ and 4kΩ resistors as this is a series circuit because current will not flow in the 8kΩ resistor as it is open. So 12V (3mA x 4kΩ) will appear across the 4kΩ resistor. We also know that current is not flowing through the 8kΩ resistor as it is open circuit, but the 8kΩ resistor is in parallel with 4k resistor. So, the same voltage (i.e. 12V) will appear across the 8kΩ resistor as 4kΩ resistor. Therefore, 12V will appear across the AB terminals. So, **VTH= 12V**

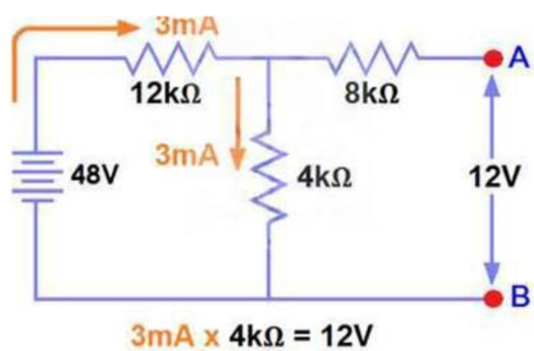


Fig.1.36

All voltage & current sources replaced by their internal impedances (i.e., ideal voltage sources short circuited and ideal current sources open circuited) as shown in fig.1.37

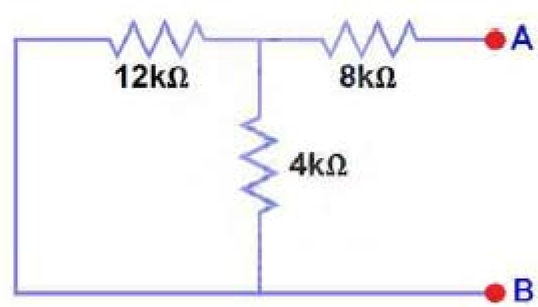


Fig.1.37

Calculate /measure the Open Circuit Resistance. This is the Thevenin Resistance (R_{TH})We have Reduced the 48V DC source to zero is equivalent to replace it with a short circuit as shown in fig.1.38 We can see that 8kΩ resistor is in series with a parallel connection of 4kΩ resistor and 12k Ω resistor. i.e.:

8kΩ + (4k Ω || 12kΩ) ….. (|| = in parallel with)

R_{TH} = 8kΩ + [(4kΩ x 12kΩ) / (4kΩ + 12kΩ)]

R_{TH} = 8kΩ + 3kΩ

R_{TH} = 11kΩ

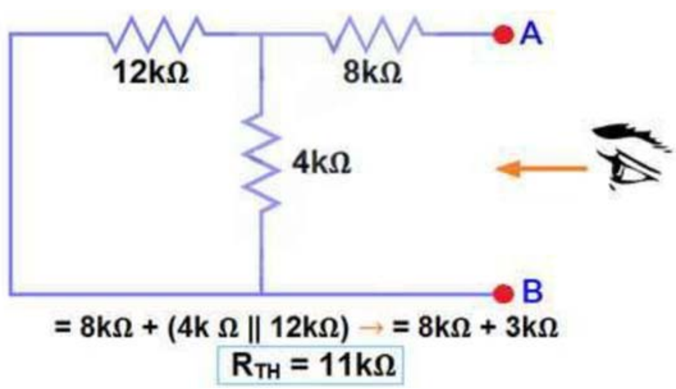
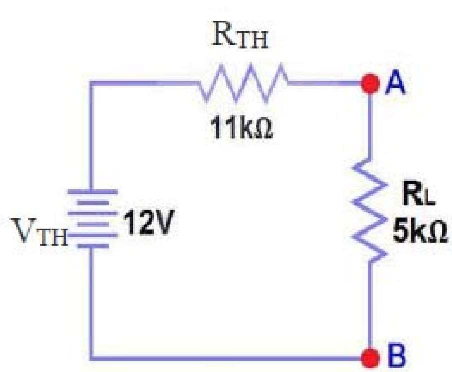


Fig. 1.38

Connect the R_{TH} in series with Voltage Source V_{TH} and re-connect the load resistor across the load terminals(A&B) as shown in fig1.38 i.e. Thevenin circuit with load resistor. This is the Thevenin's equivalent circuit



Thevenin's equivalent circuit

Fig. 1.39

Now apply Ohm's law and calculate the total load current from fig.1.39

$I_L = V_{TH} / (R_{TH} + R_L) = 12V / (11k\Omega + 5k\Omega) = 12/16k\Omega$

I_L= 0.75mA

And $V_L = I_L x R_L = 0.75mA x 5k\Omega$

V_L= 3.75V

1.4.4: Norton's Theorem

Any linear, bilateral two terminal network consisting of sources and resistors (Impedance),can be replaced by an equivalent circuit consisting of a current source in parallel with a resistance (Impedance),the current source being the short circuited current across the load terminals and the resistance being the internal resistance of the source network looking through the open circuited load terminals. Fig.1.40 shows a simple block representation of a network with several active / passive elements with the load resistance **RL** connected across the terminals **'a & b'** and fig 1.41 shows the **Norton equivalent circuit** with **IN** connected across **RN** &**RL.**

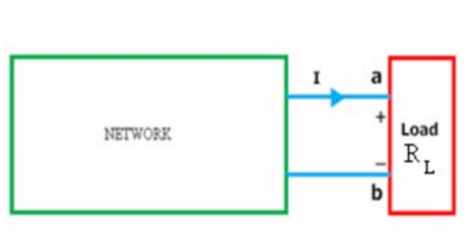


Fig.1.40

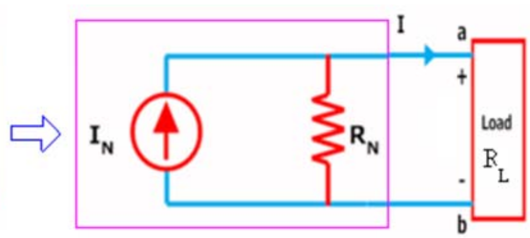


Fig.1.41

Main steps to find out IN and RN:

1. The terminals of the branch/element through which the current is to be found out are marked as say **a & b** after removing the concerned branch/element.

2. Open circuit voltage **VOC** across these two terminals and **ISC** through these two terminals are found out using the conventional network mesh/node analysis methods and they are same as what we obtained in Thevenin's equivalent circuit**.**

3. Next **Norton resistance RN** is found out depending upon whether the network contains dependent sources or not.

a) With dependent sources: $\mathbf{R_N}$ **= Voc / Isc**

b) Without dependent sources: $\mathbf{R_N}$ = Equivalent resistance looking into the concerned terminals with all voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited)

4. Replace the network with **IN** in parallel with **RN** and the concerned branch resistance across the load terminals(A&B) as shown in below Fig .1.42

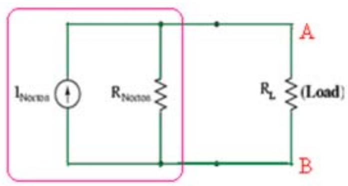


Fig.1.42

Example 1.9

Find the current through the resistance R_L (1.5 Ω) of the circuit shown in the Fig.1.43 below using Norton's equivalent circuit.

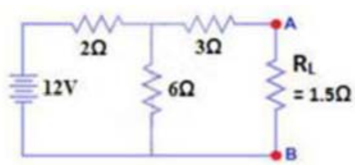


Fig.1.43

Solution

To find out the Norton's equivalent circuit we have to find out $\mathbf{I_N}$**= Isc ,** $\mathbf{R_N}$**=Voc/ Isc.** Short the 1.5Ω load resistor as shown in Fig.1.44and Calculate / measure the Short Circuit Current. This is the Norton Current (I_N).

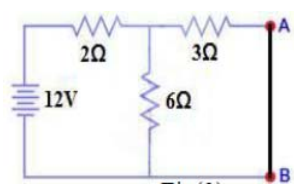


Fig.1.44

We have shorted the AB terminals to determine the Norton current, I_N. The 6Ω and 3Ω are then in parallel and this parallel combination of 6Ω and 3Ω are then in series with 2Ω. So the Total Resistance of the circuit to the Source is:-

2Ω + (6Ω || 3Ω) ….. (|| = in parallel with)

R_T = 2Ω + [(3Ω x 6Ω) / (3Ω + 6Ω)]

R_T = 2Ω + 2Ω

R_T = 4Ω

I_T = V / RT

I_T = 12V / 4Ω= 3A..

Now we have to find ISC = IN… Apply CDR… (Current Divider Rule) …

I_{SC} = I_N = 3A x [(6Ω / (3Ω + 6Ω)] = 2A.

I_{SC}= I_N = 2A.

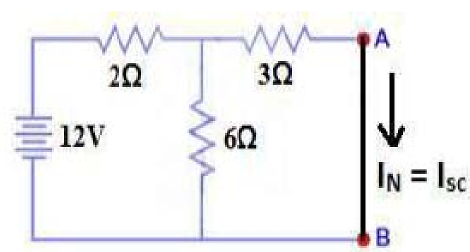


Fig. 1.45

All voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited) and Open Load Resistor. as shown in Fig.1.46

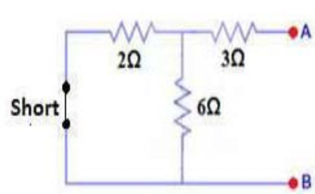


Fig.1.46

Calculate /measure the Open Circuit Resistance. This is the Norton Resistance (R_N) We have Reduced the 12V DC source to zero is equivalent to replace it with a short circuit as shown in Fig.1.46 We can see that 3Ω resistor is in series with a parallel combination of 6Ω resistor and 2Ω resistor. i.e.:

3Ω + (6Ω || 2Ω) ….. (|| = in parallel with)

$R_N = 3\Omega + [(6\Omega \times 2\Omega) / (6\Omega + 2\Omega)]$

$R_N = 3\Omega + 1.5\Omega$

$\mathbf{R_N = 4.5\Omega}$

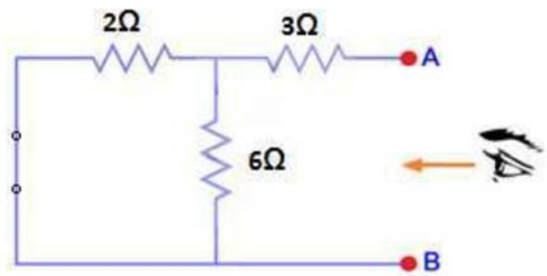


Fig. 1.47

Connect the R_N in Parallel with Current Source I_N and re-connect the load resistor. This is shown in Fig.1.48 i.e. Norton Equivalent circuit with load resistor.

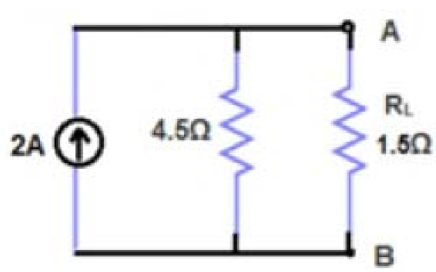


Fig.1.48

Now apply the Ohm's Law and calculate the load current through Load resistance across the terminals A&B. Load Current through Load Resistor is

$I_L = I_N \times [R_N / (R_N + R_L)]$

I_L= 2A x (4.5Ω /4.5Ω +1.5kΩ)

I_L = 1.5A **I_L = 1. 5A**

1.4.5: Maximum Power Transfer Theorem

A resistance load, being connected to a dc network, receives maximum power when the load resistance is equal to the internal resistance (Thevenin's equivalent resistance) of the source network as seen from the load terminals.

Example 1.10: Find the value of R in the circuit such that maximum power transfer takes place. What is the amount of this power?

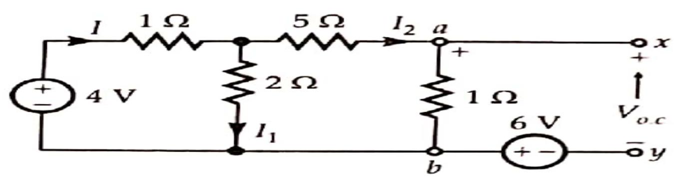


Fig.1.49

Here, $I = \frac{4V}{[\{(5+1)||2\}+1]\Omega} = \frac{4}{5/2} = \frac{8}{5}$ A

∴ $I = I\frac{2}{2+5+1} = \frac{8}{5} \times \frac{1}{4} = \frac{2}{5}$ A

The drop across a-b branch is then, $V_{a\text{-}b} = \frac{2}{5}$ x 1 = $\frac{2}{5}$ V

Obviously,

$V_{O.C} = V_{a\text{-}b} + 6V = \frac{2}{5} + 6 = \frac{32}{5}$ V

Or, $V_{O.C}$ = 6.4 V

$R_{Th} = \{1 \parallel 2 + 5\} \parallel 1 = \frac{17}{3} \parallel 1 = \frac{17}{20}\ \Omega = 0.85\ \Omega$

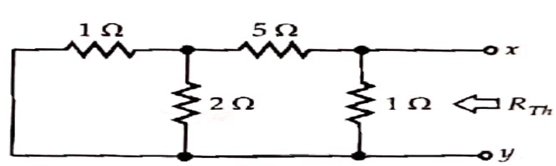


Fig. 1.50

As per maximum power transfer theorem, R = R_{Th} = 0.85 Ω, and P_{max} (max. power) = $V^2_{o.c}/4R$

$= \frac{6.4^2}{4 X 0.85} = 12W.$

Example 1.11: Assuming maximum power transfer from the source to R, find the value of this amount t of power in the circuit.

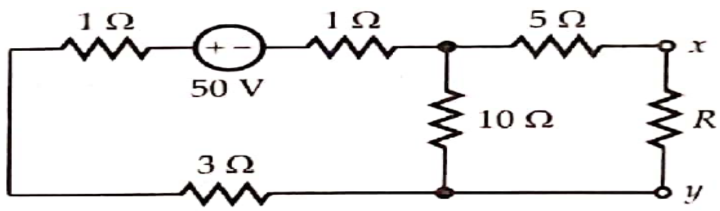


Fig. 1.51

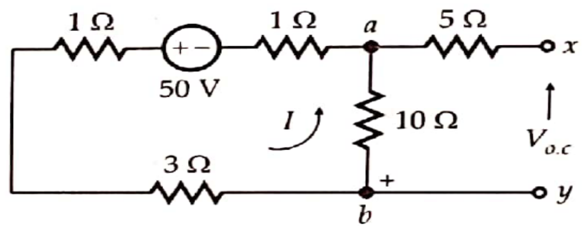


Fig. 1.52

$I = \frac{50}{15} = 3.333\ \Omega$

∴ $V_{o.c} = V_{10\ \Omega} = 3.333 \text{ x } 10 = 33.33$ V

∴ $V_{x\text{-}y} = -33.33$ V

($= V_{o.c}$)

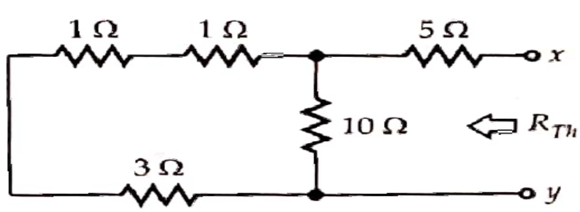


Fig. 1.53

$R_{Th} = \frac{5 \times 10}{5+10} + 5 = 8.33\ \Omega$

As per maximum power transfer theorem,

$R = R_{Th} = 8.33\ \Omega$

and $P_{max} = V^2_{o.c}/ 4R$

$= \frac{(-33.33)^2}{4 X 8.33} = 33.34$ W.

1.4.6: Compensation Theorem

In a linear time-invariant network when the resistance (R) of an uncoupled branch carrying a current (I) is changed by (ΔR), the currents in all the branches would change and can be obtained by assuming that an ideal voltage source of (Vc) has been connected [such that Vc=I (ΔR)] in series with (R+ ΔR) when all other sources in the network are replaced by their internal resistances.

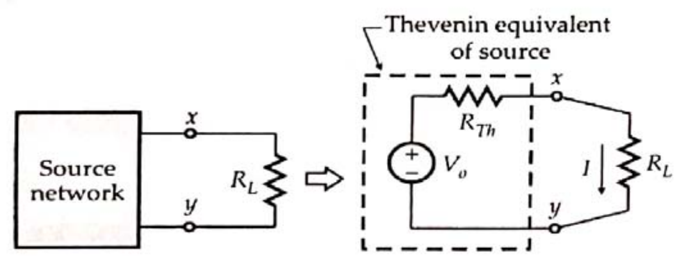


Fig. 1.54

$I = \frac{V_0}{R_{Th}+R_L}$

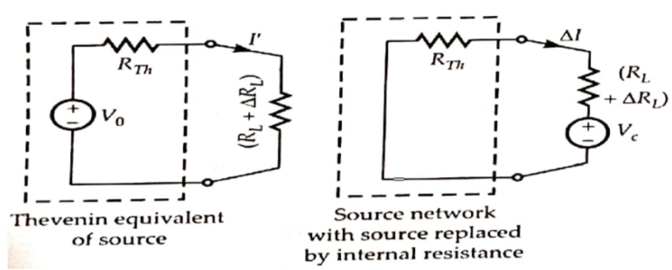


Fig.1.55

I’ = $\frac{V_0}{R_{Th}+(R_L+\Delta R_L)}$

ΔI = I’- I

= $\frac{V_0}{R_{Th}+(R_L+\Delta R_L)}$ - $\frac{V_0}{R_{Th}+R_L}$

= $\frac{V_0\{R_{Th}+R_L-(R_{Th}+R_L+\Delta R_L)\}}{(R_{Th}+R_L+\Delta R_L)(R_{Th}+R_L)}$

= -$(\frac{V_0}{R_{Th}+R_L})\frac{\Delta R_L}{R_{Th}+R_L+\Delta R_L}$

= -$\frac{I\Delta R_L}{R_{Th}+R_L+\Delta R_L}$

= $\frac{(-V_C)}{R_{Th}+R_L+\Delta R_L}$

1.4.7: Tellegen's Theorem

For any given time, the sum of power delivered to each branch of any electric network is zero. Thus, for kth branch, this theorem states that $\sum_{k=1}^{n} \quad v_k i_k = 0$; n being the number of branches, v_k the drop in the branch and i_k the through current.

Example 1.12: Check the validity of Tellegen’s theorem provided the following:

V_1 =8V, V_2 =4V, V_4 = 2V,

I_1 = 4A, I_2 = 2A, I_3 = 1A.

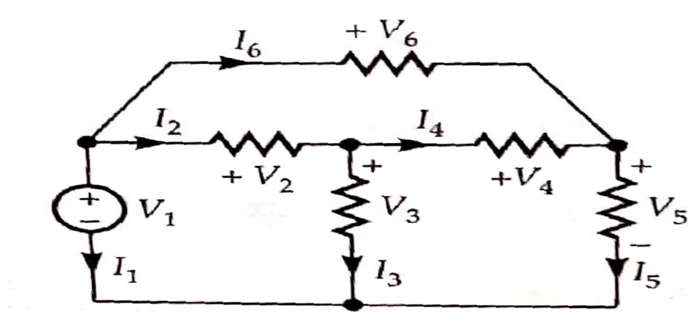


Fig.1.56

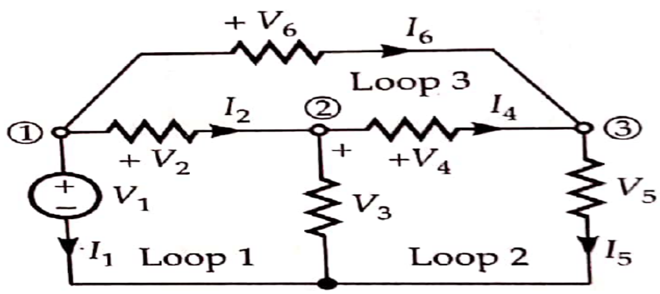


Fig.1.57

In *loop-1*, KVL yields

$-V_1 + V_2 + V_3 = 0$

Or, $V_3 = V_1 – V_2 = 8 – 4 = 4V$

Similarly, in *loop-2*, KVL yields

$-V_3 + V_4 + V_5 = 0$

or, $V_5 = V_3 – V_4$

or, $V_5 = 4 - 2 = 2$ V

and in *loop-3*, KVL yields

$-V_2 + V_6 – V_4 = 0$

Or, $V_6 = V_2 +V_4 = 4+2 = 6V$

On the other hand, at *node* (1), KCL yields

$I_1 + I_2 +I_6 =0$

$I_6 = -I_1 –I_2 = -4 – 2 = -6$ A.

Similarly, application of KCL at *node* (2) results

$I_2 = I_3 + I_4$, $I_4 = I_2 –I_3 = 2-1 = 1A$,

and at *node* (3),

$I_4 + I_6 = I_5$

Or, $I_5 = 1 + (-6) = -5A$

Therefore, Summation of powers in the branches gives

$\sum_{i=1}^{b} V_b I_b = V_1 I_1 + V_2 I_2 + V_3 I_3 + V_4 I_4 + V_5 I_5 + V_6 I_6$

$= 8 \times 4 + 4 \times 2 + 4 \times 1 + 2 \times 1 + 2 \times (-5) + 6 \times (-6)$

$= 32 + 8 + 4 + 2 - 10 - 36 = 0$

Thus, Tellegen's theorem is verified.